ABSTRACT

The aim of the paper is to address the issue of operational improvements in manufacturing firms, especially when yield losses result in a significant increase of the production cost. In a multistage production environment the firm must decide which production stage(s) should receive more “attention” in order to minimize the overall cost incurred from yield losses. Yield variability is a factor included in the decision to allocate resources to improve the production stages. The model presented in this paper considers a number of factors such as yield variability, production volumes, cost incurred from defects, and cost of improving the system. Results show that firms should focus their “attention” on production stages characterized with high yield variability since it significantly affects production cost.

KEYWORDS
Multistage Production Systems, Yield Variability, Budget Allocation, Performance Improvement

INTRODUCTION

The majority of products are manufactured in multistage stage production systems following a production sequence that depends on the nature of the product, the machines available, and the factory layout. Value is added at every production stage until the product is completed. Production sequences in multistage systems can resemble a “tree structure” or a line, referred to as a serial production system. Such systems are common in semiconductor, chemical, and food industries. In multistage systems, yield losses are inevitable due to machine and material failures during the production process, human error, and the production of products that the organization has no prior experience. Yield losses due to “process imperfections” (Hwang and Singh, 1998) result in cost increases due the scrapping of the entire product, scrapping of components, and any rework required.

The literature on multistage and serial production systems has mostly focused on production planning issues. In this line of work, the multistage production system is characterized by a number of factors, such as the yield of each production stage, the available capacity of every stage, lead times between production runs, and consumer demand. Given that these factors can be treated as deterministic or stochastic, the objective is to minimize various production-related costs in order to satisfy end demand, by deciding the production quantities to be produced at each stage. Various modeling papers have been proposed to address the above problem, some of them introducing uncertainties, such as random yields at each production stage (Lee and Yano, 1988; Akella et al., 1992; Lee, 1996; Tal and Grosfeld-Nir, 2010), random capacity due to resource unavailability (Hwang and Singh, 1998), random processing times (Grosfeld-Nir et al., 2000), and random demand (Tang, 1990; Akella et al., 1992; Denardo and Lee, 1996).
Rework has also been incorporated in the production planning problem of multistage systems, since it significantly contributes to overall cost increase (Fisher and Ittner, 1999). Such work has been carried out by Denardo and Tang (1992), Wein (1992), So and Tang (1995), Grosfeld-Nir and Gerchak (2002). Hadjinicola (2010) model a serial production system with rework using Markov chains and compute steady state costs and yields at every production stage.

The above literature on multistage production systems, basically accepts the production system's capabilities and uncertainties and thus its parameters, and attempts to determine an “optimal” mode of operation. However, one of the responsibilities of an operations manager, besides designing and operating a production system, is to further improve the system in order to reduce cost and production time and improve quality (Jacobs et al., 2009). As such, operational improvements are always a concern in manufacturing firms, especially when yield losses result in a significant increase of the production cost. In a multistage production environment the firm must decide which production stage(s) should receive more “attention” in order to minimize the cost incurred from yield losses.

The issue of improving the performance of multistage production systems has not been adequately addressed in the literature. Lee et al. (1997) present a model that computes the amount of investment needed at each production stage in a multistage system, in order to reduce the proportion of defective products and the variability of the yield at each stage. Their model is rich in terms of the parameters contributing to the decision to invest in the various stages of the multistage system. The objective is to minimize a cost function consisting of the investment, processing, set-up, and holding cost, as well as a loss cost due to the defective items. The investment made at each stage improves the proportion of defective products in an equal way at every stage, as well as variability of the yield at each stage. However, this assumption restricts the amount of investment given to each stage in a multistage system.

Hadjinicola and Soteriou (2003) address this problem and present a model that allocates limited capital resources to the various production stages with the objective to minimize the annual cost incurred from defects. Their framework considers a number of factors for resource allocation such as the mean yield of a stage, the volume of products processed at a stage, and the average cost incurred when a defect is observed. In their formulation, yield is treated as a proportion. However, in real-life manufacturing environments, yield is characterized by variability. This poses the following question: How does yield variability of the various production stages influence the decision to improve the yield of the production stages and reduce the cost incurred from defects? One would argue that production stages with highly variable yields will weigh more heavily on the cost incurred from defects.

In this paper, we present a more general modeling framework that assists management in their effort to improve the performance of its production system. We extend the work by Hadjinicola and Soteriou (2003) in the following ways: (1) yield for every product in each production stage is considered to be a random variable, (2) the families of products and their production volumes are treated differently and products processed at each production stage are not aggregated, and (3) we examine and compare two scenarios for improving the performance of the production system, a budget constrained case and the unconstrained case.

MODEL FORMULATION

Model Features

We consider a manufacturing plant consisting of dissimilar machines which we refer to as production stages. All or a subset of the production stages are used for the production of each product, which requires for its production a pre-specified sequence of stages. The modeling presented in this paper considers the annual demand for each product, as the decision to invest in improving the manufacturing facility is of strategic importance with long-term consequences.

The modeling framework uses the following notation in which we refer to both semi-completed and completed products as simply products:

- \( y_{ij} \): Yield of product \( i \) at production stage \( j \). Yield is a random variable following some distribution \( f(y_{ij}) \). It is implied that \( 0 \leq y_{ij} \leq 1 \).
- \( Y_{ij} \): Mean yield of product \( i \) at production stage \( j \).
- \( \sigma_{ij} \): Standard deviation of yield of product \( i \) at production stage \( j \).
- \( c_{ij} \): Expected cost incurred from a defect of product \( i \) observed at production stage \( j \).
- \( n_{ij} \): Annual number of products \( i \) processed at production stage \( j \). It is common in multistage production systems for production stages to process different number of products or components. For example, one stage may manufacture metallic components of a chair that will eventually be used in another stage to assemble the chair.
- \( K \): Number of different products produced by the multistage production system.
- \( N \): Number of production stages in the multistage production system.
B: Budget available for improving the yield of the production stages.

Consider an arbitrary product \( i \) at production stage \( j \) of a multistage production system operating in steady state. The yield loss of product \( i \) at stage \( j \) is equal to \( 1-y_{ij} \). The annual expected number of defects of product \( i \) to occur in stage \( j \) is given by \( \int_{-\infty}^{x} f(y_{ij}) (1-y_{ij}) n_{ij} dy_{ij} \). Therefore, the annual expected cost incurred from defects of product \( i \) at stage \( j \) is given by \( \int_{-\infty}^{x} f(y_{ij}) (1-y_{ij}) n_{ij} c_{ij} dy_{ij} \). Aggregating over all products produced at stage \( j \), the cost incurred from defects at stage \( j \) is given by:

\[
\text{Defects Cost at Stage } j = \sum_{i=1}^{K} \int_{-\infty}^{x} f(y_{ij}) (1-y_{ij}) n_{ij} c_{ij} (\sigma_{y_{ij}}) dy_{ij}.
\]

Aggregating over all production stages, the annual expected cost incurred from defects observed in all production stages is given by:

\[
\text{Total Defects Cost} = \sum_{j=1}^{N} \sum_{i=1}^{K} \int_{-\infty}^{x} f(y_{ij}) (1-y_{ij}) n_{ij} c_{ij} (\sigma_{y_{ij}}) dy_{ij}.
\]

Of course, not all products undergo operations in all production stages. To accommodate these cases, in the above cost function, we set \( f(y_{ij}) = 0 \) if product \( i \) is not produced in stage \( j \).

Investments in the various stages to improve the yield will result in a change of the yield loss for product \( i \) at stage \( j \) from its initial value of \((1-y_{ij})\) to \(\epsilon_{ij}(1-y_{ij})\). The decision variable \( \epsilon_{ij} \) represents the remaining percentage of the initial yield loss of product \( i \) at stage \( j \), after improvement in yield has been established. It follows that \( 0 \leq \epsilon_{ij} \leq 1 \) with a small values of \( \epsilon_{ij} \) implying a high yield improvement for product \( i \) at production stage \( j \). We assume that the investment will not only reduce yield loss, but it will also reduce yield variability. More specifically, the standard deviation of the yield will change from its initial value \( \sigma_{ij} \) to \( \zeta(\epsilon_{ij}) \sigma_{ij} \) where, \( 0 \leq \zeta(\epsilon_{ij}) \leq 1 \). In general, the \( \partial \zeta(\epsilon_{ij}) / \partial \epsilon_{ij} > 0 \), indicating that investments leading to small reductions in yield loss will also result in small reductions in the variability of the yield. For the sake of simplicity, we use the functional form \( \zeta(\epsilon_{ij}) = \rho \epsilon_{ij} \) where \( \rho > 0 \). Higher values of \( \rho \) imply higher reductions in yield variability, after the investment in the production stage has been implemented.

Every time a defect appears in a production stage, the firm incurs an expected cost, which results from scrapping the product, some of its components, and any rework required. We assume that \( c_{ij} \), the expected cost incurred from a defect of product \( i \) observed at production stage \( j \), is a function of the standard deviation of the yield \( \sigma_{ij} \). As such, \( c_{ij}(\sigma_{ij}) = \beta \sigma_{ij}^\delta \) which satisfies the above assumption. Higher values of the parameters \( \beta \) and \( \delta \) imply the greater the effect of yield variability on the cost incurred from defects. The cost function used above, has the advantage that after taking the logarithm of both sides, it converts into a linear equation, whose parameters \( \beta \) and \( \delta \) can be estimated using a simple linear regression model. The input to such model are the costs incurred from defects and the variability in every production stage.

To capture the phenomenon where higher yield improvement requires higher investment, we define the cost required at stage \( j \) for reducing the yield loss for product \( i \) from \((1-y_{ij})\) to \(\epsilon_{ij}(1-y_{ij})\), to be equal to \( m_{ij} \gamma_{ij} \epsilon_{ij} \). The parameter \( m_{ij} \) represents the investment required at stage \( j \) to make the yield losses for product \( i \) equal to zero, that is \( \epsilon_{ij} = 0 \). Furthermore, the parameter \( \gamma_{ij} \) represents the investment required at production stage \( j \) to reduce the yield loss of product \( i \) by one percentage point. A similar non-linear functional form of the cost needed to improve the yield at a production stage has also been used by Hadjinicola and Soteriou (2003). The choice of functions is often stylistic, however, it is important for the functions to capture the economic phenomenon, and also allow the modeler to estimate the parameters in the real-world setting.
BUDGET CONSTRAINED MODEL

Given a budget $B$ available for investments that would lead to yield improvement, the program that minimizes the total defects cost after investments in yield improvement have been established is given by:

$$
\min_{\epsilon_j} \sum_{i=1}^{K} \left\{ \sum_{j=1}^{N} \int f(y_j)(1 - y_j)\epsilon_j n_{ij}(\xi(\epsilon_j)\sigma_{ij})dy_j \right\}
$$

$$
\text{s.t. } B = \sum_{i=1}^{K} \sum_{j=1}^{N} \left[ m_{ij} - \gamma_y \epsilon_j \right].
$$

Proposition
The budget allocation that minimizes the annual expected cost incurred from defects in all production stages, will result in a change of the yield loss from $(1 - y_j)$ to $\epsilon_j^*(1 - y_j)$, where

$$
\epsilon_j^* = \alpha_j \frac{B - \sum_{i=1}^{K} \sum_{j=1}^{N} m_{ij}}{\sum_{i=1}^{K} \sum_{j=1}^{N} \gamma_y \alpha_j}, \quad i=1,...,K, \quad j=1,...,N,
$$

and

$$
\alpha_j = \left[ \frac{\gamma_y}{(1 - Y_{ij})n_{ij}\beta \sigma^\gamma (1 + \delta)\sigma^\delta} \right]^{1/\delta}
$$

Proof of Proposition: Can be obtained from authors.

DISCUSSION

Preliminary sensitivity analysis of the model solution presented in the above proposition, shows that production stages with high yield variability will receive more investments for yield improvements. The amount of money received is positively affected by the parameters $\beta$ and $\delta$. High values of these parameters imply greater effect of yield variability on the cost incurred from defects. Thus, the firm wishes to improve the yield and its variability in these cost incurring stages. Furthermore, stages characterized by high values of $\rho$, meaning higher reductions in yield variability after the investment in the production stage has been implemented, receive significant investments for yield variability. The solution of the model can provide guidelines or regions of investments to improve the yield in real life manufacturing facility. Furthermore, the model presented in this paper can be complemented with an unconstrained model for yield and yield variability improvements. Comparison of these two cases can assist management to better allocate its resources to improve the manufacturing system.

REFERENCES


