POSSIBILITY APPROACH FOR SOLVE THE DATA ENVELOPMENT ANALYTICAL HIERARCHY PROCESS (DEAHP) WITH FUZZY JUDGMENT SCALES

by

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ABSTRACT

The data envelopment analytical hierarchy process (DEAHP) is a type of the analytical hierarchy process (AHP), which used the concept of data envelopment analysis (DEA) for generating LW from the judgment matrices and aggregating them to be final weight (FW) in AHP. In general, the judgment scales in AHP is built based on decision maker. It is unrealistic to expect that the decision maker has complete information of all aspects of the problem. Thus, the fuzzy judgment scales will be firstly modeled in this paper. Sometime, there are many decision makers. It is a caused of randomness of judgment scales. In this case, the fuzzy judgment scales are treated to be the random variable (RV), which is the fuzzy random variable (FRV). However, the traditional DEAHP model has often treated judgment scales as being crisp deterministic. Then, it cannot be solved by the standard linear programming concept. In this paper, the concept of chance-constrained programming (CC) and the possibility approach (PA) are chosen to convert the fuzzy stochastic data envelopment analytical hierarchy process (FSDEAHP) model into equivalent crisp deterministic DEAHP (E-CD-DEAHP) model. It can be solved by traditional mathematical programming.

KEYWORDS
Analytical Hierarchy Process, Chance-Constrained Programming, Data Envelopment Analysis, Fuzzy Set Theory, Possibility Approach

INTRODUCTION

The analytic hierarchy process (AHP) was firstly proposed by Saaty (1980). It is an effectively decision-making analysis tool that uses hierarchical structures to solve the complicated and unstructured decision problems. In General, there are three steps in classical AHP. Step 1, structuring the hierarchy of criteria and alternatives for evaluation. For more complex problems, the sub-criteria can be added to link between criteria and alternatives. Step 2, judgment matrix or pair-wise comparison matrix of criteria and alternatives in each criteria are constructed. Elements, which are criteria and alternative, are compared pair-wise and judgments on comparative attractiveness of elements are captured using the discrete scale of judgments scale for pair-wise comparisons. Let a_{ij} for all i and j be elements in judgment matrix A. If the transitivity property holds, i.e., a_{ij} = a_{ik} x a_{kj}, for all the entries of the matrix, then the matrix is said to be consistent. If the property does not hold for all the entries, the level of inconsistency can be captured by a measure called consistency ratio. Based on eigenvector method, let \( \phi_{\text{max}} \) be the maximum eigenvalue of A, which is always greater than or equal to n for positive reciprocal matrices and is equal to n if and only if A is a consistent matrix. The consistency index (CI), which is used to measure the variance of the error incurred in estimating the matrix A, is calculated by CI = (\( \phi_{\text{max}} – 1 \))/(n – 1). To measure how a given matrix compares to a purely random matrix in term of CI, the ratio of the CI to the random index (RI), which is called a consistency ratio (CR), was computed. Step 3, calculating local weights (LW) and final weight of alternatives (FW). In classical AHP, LWs, which is yield priorities of each criteria and alternative, can be calculated by the eigenvector method (EVM). For FWs, the priorities of the alternatives by criteria can be synthesized based on hierarchical arithmetic aggregation. Since AHP has unique advantages when important elements of the decision are difficult to quantify or compare, or where communication among team members is impeded by their different specializations, terminologies, or perspectives. It has successfully been applied to many actual decision situations in many areas such as selection, evaluation, planning and development, decision making, forecasting, and so on. There was numerous research papers on decision-making based on AHP have been conducted (see Shim 1989; Vaidya and Kumar, 2006, for a bibliographical research on AHP applications).
**BACKGROUND**

Data Envelopment Analytical Hierarchy Process

The data envelopment analytical hierarchy process (DEAHP) was first introduced by Ramanathan (2006). It is the AHP which uses the DEA concept (Charnes et al., 1978) for generating LWs from each judgment matrix and aggregating them to be FWs. Focused on LWs, let A be judgment matrix of size n x n (compare n alternatives) and \( a_{ij} \) be entities of A, thus there are 1 dummy input, n outputs and n DMUs of the DEAHP model. Based on the first DEA model, the input oriented CCR model for LW and its duel problem are the following linear programming (LP) problem.

\[
\text{(DEAHP-CCR-I-LW) Max } \theta = \sum_{i=1}^{n} u_i a_{io} \\
\text{Subject to } \nu = 1 \\
\sum_{i=1}^{n} u_i a_{ij} - \nu \leq 0 \text{ for } j = 1, \ldots, n \\
u, \nu \geq 0,
\]

\[
\text{(DEAHP-DCCR-I-LW) Min } \theta \\
\text{Subject to } \theta - \sum_{j=1}^{n} \lambda_j \geq 0 \\
a_{io} - \sum_{j=1}^{n} \lambda_j a_{ij} \leq 0; i = 1, \ldots, n \\
\theta \text{ Unrestricted, } \lambda_j \geq 0,
\]

where \( u_i, \nu, \theta \text{ and } \lambda_j \) for all \( i, j \) are decision variables of primal and dual problem, respectively.

To aggregate LWs to get FW of each alternative based on the concept of DEA, the importance values of criteria are automatically generated as the value of multipliers using LP. Let \( \theta_{ki} \) be LW of alternative i in criteria k for \( i = 1, \ldots, n \) and \( k = 1, \ldots, m \) from DEAHP-LW model. The input oriented CCR model for FW of alternatives and its duel problem are the following linear programming (LP) problem.

\[
\text{(DEAHP-CCR-I-FW) Max } \phi = \sum_{k=1}^{m} w_k \theta_{ko} \\
\text{Subject to } x = 1 \\
\sum_{k=1}^{m} w_k \theta_{ki} - x \leq 0 \text{ for } i = 1, \ldots, n \\
w_i, x \geq 0,
\]

\[
\text{(DEAHP-DCCR-I-FW) Min } \phi \\
\text{Subject to } \phi - \sum_{i=1}^{n} \gamma_i \geq 0 \\
\theta_{ko} - \sum_{i=1}^{n} \gamma_i \theta_{ki} \leq 0; k = 1, \ldots, m \\
\phi \text{ Unrestricted, } \gamma_i \geq 0,
\]

where \( w_i, x, \phi \text{ and } \gamma_i \) for all \( i, k \) are decision variables of primal and dual problem, respectively.

**Chance-Constrained Programming**
CC is a kind of stochastic optimization approaches. It is suitable for solving optimization problems with random variables included in constraints and sometimes in the objective function as well. The constraints are guaranteed to be satisfied with a pre-specified minimum probability or confidence level at the optimal solution found (Charnes and Cooper, 1959). Let \( c_j, \hat{a}_{ij} \) and \( \hat{b}_i \) be random variables and \( x_j \) be decision variables for \( i = 1, \ldots, n \) and \( j = 1, \ldots, m \), then the stochastic programming is modeled in the following equations.

\[
\text{Min } Z = \sum_{j=1}^{n} \hat{c}_j x_j \tag{17}
\]

\[
\text{St. } \sum_{j=1}^{n} \hat{a}_{ij} x_j \leq \hat{b}_i \text{ for } i = 1, \ldots, m \tag{18}
\]

\[
\forall x_j \geq 0 \tag{19}
\]

The stochastic programming in (9)-(11) can be transformed to be the following probability linear programming.

\[
\text{Min } Z = f \tag{20}
\]

\[
\text{St. } \Pr\left( \sum_{j=1}^{n} \hat{c}_j x_j \leq f \right) \geq 1 - \alpha \tag{21}
\]

\[
\Pr\left( \sum_{j=1}^{n} \hat{a}_{ij} x_j \leq \hat{b}_i \right) \geq 1 - \alpha_i \text{ for } i = 1, \ldots, m \tag{22}
\]

\[
f, x_j \geq 0, \forall x_j \tag{23}
\]

where “\( \Pr \)” means probability, \( 1 - \alpha \) and \( 1 - \alpha_i \) are pre-specified minimum probability for all \( i \) and \( f \) is artificial variable. Subsequently, some researchers adopt these theoretical results in the field of stochastic DEA (Olesen and Petersen, 1995; Cooper et al., 1996; Cooper et al., 1998; Li, 1998; Sueyoshi, 2000; Cooper et al., 2002).

**Possibility Approach**

Possibility theory in the context of the fuzzy set theory was introduced by Zadeh (1978) which was dealing with non-stochastic imprecision and vagueness, good references on possibility theory can be found in Dubois and Prade (1980) and Zimmermann (1996). Suppose that \((\Theta_i, P(\Theta_i), \pi_i)\) for \( i = 1, \ldots, n \) is a possibility space with \( \Theta_i \) being the nonempty set of interest, \( P(\Theta_i) \) is the collection of all subsets of \( \Theta_i \), and \( \pi_i \) is the possibility measure from \( P(\Theta_i) \) to \([0, 1]\), then \( \pi(\emptyset) = 0 \) and \( \pi(\Theta_i) = 1 \), and \( \pi(\cup A_i) = \sup \{ \pi(A_i) \} \) with each \( A_i \in P(\Theta_i) \). Let \( \tilde{\xi} \) be fuzzy variable as a real-valued function defined over \( \Theta \), therefore the membership function of \( \tilde{\xi} \) is given by

\[
\mu_{\tilde{\xi}}(s) = \pi(\{ \theta_i \in \Theta / \tilde{\xi}(\theta_i) = s \}) = \sup_{\theta_i \in \Theta} \{ \pi(\{ \theta_i \})/\tilde{\xi}(\theta_i) = s \}, \forall s \in \mathcal{R}. \tag{24}
\]

Let \((\Theta, P(\Theta), \pi)\) be a product possibility space such that \( \Theta = \Theta_1 \times \ldots \times \Theta_n \), then

\[
\pi(A) = \min \{ \pi_i(A_i) / A = A_1 \times \ldots \times A_n, A_i \in P(\Theta_i) \} . \tag{25}
\]

To compare fuzzy variables (Dubois and Prade, 1980), let \( \tilde{a}_j, \ldots, \tilde{a}_n \) be fuzzy variables and \( f_j : \mathcal{R}^n \rightarrow \mathcal{R} \) be a real-valued function for \( j = 1, \ldots, m \). The possibility measure of fuzzy event is given by

\[
\pi(f_j(\tilde{a}_1, \ldots, \tilde{a}_n) \leq 0) = \sup_{s_1, \ldots, s_n \in \mathcal{R}} \{ \min \{ \mu_{\tilde{a}_i}(s_i) / f_j(s_1, \ldots, s_n) \leq 0 \} . \tag{26}
\]

Possibility measures are adopted to prove Lemma 1 for solving the fuzzy multiplier form of input-oriented CCR (FCCR-1) model by Lertworasirikul et al. (2003).
Lemma 1. Let $\tilde{a}_1, \ldots, \tilde{a}_n$ be fuzzy variables with normal and convex membership functions and $b$ be a crisp variable. Let $(\bullet)_{\tilde{a}_i}^L$ and $(\bullet)_{\tilde{a}_i}^U$ denote the lower and upper bounds of the $\alpha$-level set of $\tilde{a}_i$ for $i = 1, \ldots, n$. Then, for any given possibility levels $\alpha_i$, $\alpha_2$, and $\alpha_3$ with $0 \leq \alpha_i, \alpha_2, \alpha_3 \leq 1$

(i) \[ \pi(\tilde{a}_i + \Lambda + \tilde{a}_n \leq b) \leq \pi(\tilde{a}_i)^L_{\alpha_1} + \Lambda + (\tilde{a}_n)^L_{\alpha_2} \leq b, \]

(ii) \[ \pi(\tilde{a}_i + \Lambda + \tilde{a}_n \geq b) \geq \pi(\tilde{a}_i)^U_{\alpha_2} + \Lambda + (\tilde{a}_n)^U_{\alpha_3} \geq b, \]

(iii) \[ \pi(\tilde{a}_i + \Lambda + \tilde{a}_n = b) \geq \pi(\tilde{a}_i)^L_{\alpha_3} + \Lambda + (\tilde{a}_n)^L_{\alpha_3} \leq b \]

and $(\tilde{a}_i)^U_{\alpha_3} + \Lambda + (\tilde{a}_n)^U_{\alpha_3} \geq b$.

**FUZZY STOCHASTIC DEAHP FOR LOCAL WEIGHTS**

Due to the complexity and uncertain decision problems, the pair-wise comparison matrices may be unrealistic to expect that the decision maker have either complete information or a full understanding of all aspects of the problem, which are represented as crisp numbers. Therefore, the fuzzy scales are required. In this paper, the triangular fuzzy number, which is the most basic fuzzy number, is chosen to be fuzzy scales for DEAHP model. Let fuzzy numbers $\tilde{c}_i = ((\tilde{c}_i)^L_0, (\tilde{c}_i)^L_1, (\tilde{c}_i)^U_0)$ for $i = 1, \ldots, n$ be triangular fuzzy numbers, $\mu_{\tilde{c}_i}(t)$ be the membership function of $\tilde{c}_i$, and $(\tilde{c}_i)^L_0$ and $(\tilde{c}_i)^U_0$ respectively be the lower and upper bounds of the $\alpha$-level set of $\tilde{c}_i$ (see Figure 1). Then the closed crisp interval of $\tilde{c}_i$ is defined by

\[
(1 - \alpha)(\tilde{c}_i)^L_0 + \alpha(\tilde{c}_i)^U_0 \leq t \leq (1 - \alpha)(\tilde{c}_i)^L_1 + \alpha(\tilde{c}_i)^U_1.
\] (27)

In this paper, the minimum scale 1 is set to be crisp value, fuzzy judgments scale 2-8 are set to be symmetry triangular fuzzy number with the lower and upper spreads = 1, thus fuzzy scale 2-8 can be respectively rewritten in terms of $\alpha$-level set as follow; $\tilde{2} = [\alpha + 1, 3 - \alpha], \tilde{3} = [\alpha + 2, 4 - \alpha], \ldots, \tilde{8} = [\alpha + 7, 9 - \alpha]$, and maximum scale $\tilde{9}$ be triangular fuzzy number with the lower spread = 1, thus scale $\tilde{9}$ can be rewritten in terms of $\alpha$-level set as $\tilde{9} = [\alpha + 8, 9]$. Since scale $\tilde{2}$ - $\tilde{9}$ are positive fuzzy number, therefore $1/\tilde{2}$ - $1/\tilde{9}$ can be calculated by extended division operator of fuzzy arithmetic (Zimmermann, 1990), e.g.

\[
1/(\tilde{A}) = [\min\{1/(\tilde{A})^L_{\alpha}, 1/(\tilde{A})^U_{\alpha}\}, \max\{1/(\tilde{A})^L_{\alpha}, 1/(\tilde{A})^U_{\alpha}\}]
\] (28)

where $\tilde{A}$ is positive fuzzy number. Therefore, $1/\tilde{2} = [1/(3 - \alpha), 1/(\alpha + 1)], \ldots, 1/\tilde{8} = [1/(9 - \alpha), 1/(\alpha + 7)],$ and $1/\tilde{9} = [1/9, 1/(\alpha + 8)]$.

**FIGURE 1**

TRIANGULAR FUZZY NUMBER AND ITS MEMBERSHIP FUNCTION

![Triangular Fuzzy Number and Its Membership Function](image-url)
If there are greater than one decision-maker, the judgment scales must have randomness from sampling. In this case, the fuzzy random scales $\tilde{a}_{ij}$ for all $i$ and $j$, which are measurable function from a probability space to the set of fuzzy variables (Kwakernaak, 1978), are required in the DEAHP under the random and vague decision making. It is called fuzzy stochastic DEAHP (FSDEAHP).

$$(FSDEAHP-CCR-I-LW) \quad \text{Max} \; \theta = \sum_{i=1}^{n} u_i \tilde{a}_{io} \tag{29}$$

Subject to $\sum_{i=1}^{n} u_i \tilde{a}_{ij} \leq 1$ for $j = 1, \ldots, n$ \hspace{1cm} (30)

$u_i \geq 0$ \hspace{1cm} (31)

$$(DEAHP-DCCR-I-LW) \quad \text{Min} \; \theta \tag{32}$$

Subject to $\theta - \sum_{j=1}^{n} \tilde{\lambda}_j \geq 0 \tag{33}$

$\tilde{a}_{io} - \sum_{j=1}^{n} \tilde{\lambda}_j \tilde{a}_{ij} \leq 0; \; i = 1, \ldots, n \tag{34}$

$\theta$ Unrestricted, $\tilde{\lambda}_j \geq 0$, \hspace{1cm} (35)

Since traditional DEAHP model is a type of LP model thus the crisp deterministic assumption is necessary. This assumption is caused of LW of FDEAHP model cannot be solved by standard LP. In this paper, the transformation method for transform the FSDEAHP model to be the equivalent crisp deterministic DEAHP models (E-CDDEAHP) based on CC and PA are shown. However, since the steps of transformation of FSDEAHP-CCR-I-LW be similar to DEAHP-DCCR-I-LW, and there are three important reasons for solving the envelopment model instead of solving its primal model. First, the number of DMUs ($n$) is larger than the number of inputs and outputs ($m + s$) and hence it takes more time and larger memory to solve primal problem or multiplier model with $n$ constraints than to solve the envelopment model with $m + s$ constraints. Second, the activities of inefficient DMU cannot be improved because the reference set and max-slack solution cannot be found in multiplier models. Finally, the interpretations of envelopment models are more straightforward than these of multiplier models (Cooper et al. 2000). Therefore, the FSDEAHP-DCCR-I-LW model is used to show this transformation method.

**EQUIVALENT CRISP DETERMINISTIC DEAHP FOR LWs**

Focused on FSDEAHP-DCCR-I-LW, since there is only (34) is the fuzzy stochastic constrains. Then (34) must be covert to be the probability and possibility constrains in the first, that is,

$$(CC-FSDEAHP-DCCR-I-LW) \quad \text{Min} \; \theta \tag{36}$$

Subject to $\theta - \sum_{j=1}^{n} \tilde{\lambda}_j \geq 0 \tag{37}$

$$\pi \left\{ \text{Pr} \left\{ \tilde{a}_{io} - \sum_{j=1}^{n} \tilde{\lambda}_j \tilde{a}_{ij} \leq 0 \right\} \geq (1 - \alpha_i) \right\} \geq \beta_i \; \text{for} \; i = 1, \ldots, n \tag{38}$$

$\theta$ Unrestricted, $\tilde{\lambda}_j \geq 0$, \hspace{1cm} (39)

where “$Pr$” means probability and $1 - \alpha_i$ is a pre-specified minimum probability. “$\pi$” means possibility and $\beta_i$ are pre-specified acceptable levels of possibility.
Let \( \rho_i \) for \( i = 1, \ldots, n \) be slacks which can be inserted in inequality outside braces to achieve equality, and \( s_i^+ \) be positive variables, therefore

\[
\pi \left\{ \Pr \left\{ \tilde{z}_{a_{io}} - \sum_{j=1}^{n} \lambda_j \tilde{z}_{a_{ij}} \leq 0 \right\} = (1 - \alpha_i) + p_i \right\} \geq \beta_i \text{ for } i = 1, \ldots, n, \tag{40}
\]

and

\[
\pi \left\{ \Pr \left\{ \tilde{z}_{a_{io}} - \sum_{j=1}^{n} \lambda_j \tilde{z}_{a_{ij}} \leq -s_i^+ \right\} = 1 - \alpha_i \right\} \geq \beta_i \text{ for } i = 1, \ldots, n. \tag{41}
\]

The fuzzy random scales are assumed to be normal distributed with mean zero and variance one, therefore (41) is normalized by

\[
z_i = \frac{\tilde{z}_{a_{io}} - \sum_{j=1}^{n} \lambda_j \tilde{z}_{a_{ij}} - \mu_{\tilde{a}_{io}} + \mu_{\tilde{a}_{ij}}}{\sqrt{[1, -\lambda]^T \text{Cov}[1, -\lambda]}}. \tag{42}
\]

where \([1, -\lambda] = (1, -\lambda_1, \ldots, -\lambda_n)^T\) is the vector of decision variable and \(\text{Cov}\) is the variance-covariance matrix of fuzzy output random variables for the DMU \(i\).

Let the left hand side term inside “Pr” braces of (41) denoted by \(z_i\) for \(i = 1, \ldots, n\). Therefore, (41) can be reformulated to be

\[
\pi \left\{ \Pr \left\{ \left\{ -s_i^+ - \mu_{\tilde{a}_{io}} + \mu_{\tilde{a}_{ij}} \right\} \right\} \right\} = 1 - \alpha_i \right\} \geq \beta_i. \tag{43}
\]

So,

\[
\pi \left\{ \sum_{j=1}^{n} \lambda_j \text{E}(\tilde{a}_{ij}) - \text{E}(\tilde{a}_{io}) - s_i^+ = (\Phi^{-1}(1 - \alpha_i))\sqrt{[1, -\lambda]^T \text{Cov}[1, -\lambda]} \right\} \geq \beta_i \tag{44}
\]

where \(\Phi\) represents the normal cumulative distribution function and \(\Phi^{-1}\) is its inverse. Since (44) is expressed by the expected value and the quadratic terms of variance-covariance matrices, solving the FDDCCR model is a non-trivial task. In this paper, the linearization approach to obtain a linear deterministic equivalent model, which was introduced by Cooper et al. (1998) is used. Let \(\tilde{\alpha}_{io}\) and \(\tilde{\alpha}_{ij}\) represent fuzzy means, and \(\tilde{\sigma}_{io}\) and \(\tilde{\sigma}_{ij}\) represent fuzzy standard deviations of fuzzy scales. Let \(\xi_{io}\) and \(\xi_{ij}\) be error structures which are assumed to be standard normal distributed. Then the data structures of fuzzy scales can be written as \(\tilde{\alpha}_{io} = \bar{a}_{io} + \tilde{\sigma}_{io} \xi_{io}\) and \(\tilde{\alpha}_{ij} = \bar{a}_{ij} + \tilde{\sigma}_{ij} \xi_{ij}\). Therefore, fuzzy expected value and variance are given by

\[
\text{E}(\tilde{a}_{io}) = \bar{a}_{io}, \quad \text{E}(\tilde{a}_{ij}) = \bar{a}_{ij}, \quad \text{Var}(\tilde{a}_{io}) = \tilde{\sigma}_{io}^2 \quad \text{and} \quad \text{Var}(\tilde{a}_{ij}) = \tilde{\sigma}_{ij}^2. \tag{45}
\]
To simplify FSDEAHP model, the correlations of all entities are assumed to be 1, then variance and covariance matrix of fuzzy scales are given by,

$$
\text{Cov} = \begin{bmatrix}
\tilde{\sigma}_{i0}^2 & \tilde{\sigma}_{i0} \tilde{\sigma}_{il} & \tilde{\sigma}_{i0} \tilde{\sigma}_{in} \\
\tilde{\sigma}_{il} \tilde{\sigma}_{i0} & \tilde{\sigma}_{il}^2 & \tilde{\sigma}_{il} \tilde{\sigma}_{in} \\
\tilde{\sigma}_{in} \tilde{\sigma}_{i0} & \tilde{\sigma}_{in} \tilde{\sigma}_{il} & \tilde{\sigma}_{in}^2 \\
\end{bmatrix}.
$$

(46)

So, the term of fuzzy standard deviation in (44) can be reformulated to

$$
\sqrt{[1, -\lambda]^T \text{Cov}[1, -\lambda]} = \sum_{j=1}^{n} \lambda_j \tilde{\sigma}_{ij} - \tilde{\sigma}_{i0}
$$

(47)

Substituting of $\kappa = \Phi^{-1}(1-\alpha_i)$ and the term of fuzzy standard deviation in (47) into (44), then the equivalent fuzzy deterministic DCCR model (E-FDDEAHP-DCCR-I-LW) in terms of fuzzy LP problem can be formulated as follows,

(E-FDDEAHP-DCCR-I-LW) Min $\theta$

Subject to $\theta - \sum_{j=1}^{n} \lambda_j \geq 0$

$$
\pi \left( \tilde{\alpha}_{io} - \kappa \tilde{\sigma}_{io} - \sum_{j=1}^{n} \lambda_j \tilde{\alpha}_{ij} - \kappa \tilde{\sigma}_{ij} \right) \leq 0 \text{ for } i = 1, \ldots, n
$$

(50)

$\theta$ Unrestricted, $\lambda_j \geq 0$.

(51)

From E-FDDEAHP-DCCR-I-LW model, the output constrains in (50) are the fuzzy constrains, Lemma 1 is used to convert them to be the equivalent crisp deterministic constrains as given by

$$
\left( \tilde{\alpha}_{io} - \kappa \tilde{\sigma}_{io} - \sum_{j=1}^{n} \lambda_j \tilde{\alpha}_{ij} - \kappa \tilde{\sigma}_{ij} \right)_{\beta_i}^{L} \leq 0.
$$

(52)

Substituting of the equivalent crisp deterministic constrains in (52) into (50), then the equivalent crisp deterministic DCCR model (E-CDDEAHP-DCCR-I-LW) in terms of LP problem can be formulated as follows,

(E-CDDEAHP-DCCR-I-LW) Min $\theta$

Subject to $\theta - \sum_{j=1}^{n} \lambda_j \geq 0$

$$
\left( \tilde{\alpha}_{io} \right)_{\beta_i}^{L} - \kappa (\tilde{\sigma}_{io})_{\beta_i}^{U} - \sum_{j=1}^{n} \lambda_j \left( \tilde{\alpha}_{ij} \right)_{\beta_i}^{U} - \kappa (\tilde{\sigma}_{ij})_{\beta_i}^{L} \leq 0 \text{ for } i = 1, \ldots, n
$$

(55)

$\theta$ Unrestricted, $\lambda_j \geq 0$.

(56)

The lower and upper of fuzzy means and standard deviations in (55) are the crisp value. It can be calculated by the extension principal (Zimmermann, 1996; Klir et al., 1997). Thus, the LP problem in (53)-(56) are satisfied the crisp deterministic assumption. To find the optimal relative efficiency $\theta^*$ of each DMU at pre-specified minimum probability for all $\alpha$-level set $\beta_i$, the E-CDDEAHP-DCCR-I-LW can be solved by the standard mathematical programming method.
FUZZY DEAHP AND EQUIVALENT CRISP DEAHP FOR FWS

Since the optimal relative efficiency or LWs, which are solved based on the E-CDEAHP-DCCR-I-LW at all \( \beta_i \), is the fuzzy weights. So, the DEAHP-DCCR-I-FW is in a form of fuzzy LP model. Let \( \tilde{\theta}_{ki} \) be a fuzzy LW of alternative \( i \) in criteria \( k \) for \( i = 1, \ldots, n \) and \( k = 1, \ldots, m \) from DEAHP-LW model. The fuzzy DEAHP-DCCR-I-FW are following fuzzy programming problem.

\[
\text{(FDEAHP-DCCR-I-FW)} \quad \text{Min } \varphi
\]

Subject to

\[
\varphi - \sum_{i=1}^{n} \gamma_i \geq 0 \tag{57}
\]

\[
\tilde{\theta}_{ko} - \sum_{i=1}^{n} \gamma_i \tilde{\theta}_{ki} \leq 0; \ k = 1, \ldots, m \tag{58}
\]

\( \varphi \) Unrestricted, \( \gamma_i \geq 0 \), \( \gamma_i \) unrestricted.

Due to fuzzy constrains (59), it can be converted to be possibility constrains by

\[
\pi \left( \tilde{\theta}_{ko} - \sum_{i=1}^{n} \gamma_i \tilde{\theta}_{ki} \leq 0 \right) \geq \beta_i \text{ for } k = 1, \ldots, m. \tag{61}
\]

Lemma 1 is used to convert them to be the equivalent crisp constrains as given by

\[
\left( \tilde{\theta}_{ko} - \sum_{i=1}^{n} \gamma_i \tilde{\theta}_{ki} \right)_{\beta_i}^{L} \leq 0 \text{ for } k = 1, \ldots, m. \tag{62}
\]

Substituting of the equivalent crisp constrains in (62) into (59), then the equivalent crisp DEAHP model (E-CDEAHP-DCCR-I-FW) in terms of LP problem can be formulated as follows,

\[
\text{(E-CDEAHP-DCCR-I-FW)} \quad \text{Min } \varphi
\]

Subject to

\[
\varphi - \sum_{i=1}^{n} \gamma_i \geq 0 \tag{63}
\]

\[
(\tilde{\theta}_{ko})_{\beta_i}^{L} - \sum_{i=1}^{n} \gamma_i (\tilde{\theta}_{ki})_{\beta_i}^{L} \leq 0; \ k = 1, \ldots, m \tag{64}
\]

\( \varphi \) Unrestricted, \( \gamma_i \geq 0 \).

The optimal relative efficiency \( \varphi^* \) or FWs can be found by standard LP solving at all all \( \alpha \)-level set \( \beta_i \). The membership function of FWs can be predicted by the regression technique, \( \varphi \) at each the \( \alpha \)-level set is response and sample of \( \beta_i \) are the predictors.
CONCLUSION

This paper has presented the equivalent crisp deterministic DEAHP model as a way to solve DEA with fuzzily imprecise and probabilistically uncertainty in the judgment scales of DEAHP. The concept of chance-constrained programming and the possibility approach are used to converted FSDEAHP for generate local weights, which is not in accordance with crisp deterministic assumption, to be the form of a general LP model or E-CDDEAHP-LW. The results of E-CDDEAHP-LW model are the fuzzy local weights, so the DEAHP for aggregate local weights to be the final weight of each alternative is in the form of the fuzzy LP model. The possibility approach is chosen to covert the fuzzy model to be the equivalent crisp DEAHP model. The results of the E-CDEAHP-FW are the fuzzy final weight at each the $\alpha$-level set, and its membership function can be predicted be the regression technique.

REFERENCES


